Evaluating Vibrator Spacing Including Mutual Admittance Interaction Effects
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Summary
There is a long-standing theoretical basis for the interactions between vibrators based on the concepts of mutual admittance. These theoretical interactions particularly indicate significant changes in the amount of Rayleigh wave energy generated as vibrator separation changes. These changes are ‘beyond’ classical linear superposition source array theory. A simple field test geometry is utilized to assess vibrator interactions. This tests vibrator separation distances in a manner separate from classical linear superposition source array theory. Field data indicate significant variations generally in the manner of those predicted by theory.

Introduction
Ongoing development of Vibroseis technology has taken place over several decades. This has included significant advances in control systems, harmonic reduction, increased bandwidth, increased output, and other improvements that have enhanced efficiency and data quality. Additionally, over time there have been significant variations in the typical practice of how vibrators are deployed singly or in arrays.

One aspect of Vibroseis technology that does not seem to have received the attention it deserves is the expected far-field allocation of energy in compressional body waves, shear body waves, and Rayleigh waves as a function of the vibrator spacing due to mutual admittance effects beyond classical linear superposition source array concepts.

Theoretical Background
Miller and Pursey (1955), Tan (1985) and others have developed a theoretical basis for the expected far-field energy in compressional and shear body waves, and Rayleigh waves for vertical force sources, such as vertical vibrators or weight drop. These theories have indicated that there are significant effects that are not described by classical linear superposition source array theory. Total energy emitted does not vary linearly with the number of vibrators. For n vibrators at close spacing relative to wavelengths, the total energy increases as n**2. The amount of P, S, and Rayleigh wave energy each vary in a separate manner depending on the spacing between vibrators. Particularly, Rayleigh wave energy is a strong function of vibrator spacing. Much of this theoretical analysis has been done with classical linear superposition and the non-superposition mutual admittance effects combined.

In broad general concept, power emitted is the product of force or pressure and particle velocity, including mutual admittance cross-products. The mutual admittance effects in the near field include the products of force due to vibrator A times particle velocity due to vibrator B, and vice versa. In the far field, particle velocity is proportional to the square root of power, related by specific impedance/admittance at the receiver.

Mutual admittance depends on the relative geometry (spacing) between vibrators, and on the rock properties. It does not depend on the force levels, i.e. is not a non-linear effect of ‘overdriving’ the elastic earth.

Early Developments
Perhaps the earliest publication in the geophysical literature concerning mutual admittance effects between vibrators was Cassand and Lavergne (1971, 1966). Figure 2 here is from that publication; it is still a good representation of the results of these interactions. The x axis of this figure is vibrator spacing in terms of compressional wavelength. The left end represents vibrators “on top” of each other. The right end value of two*πi represents vibrators separated by one compressional wavelength. This is for a Poisson solid. The y axis is total energy emitted on a linear scale. Total energy is summed over all azimuths and diving angles for the hemisphere under the source array. The three shaded zones represent P, S, and Rayleigh energy.

Note that the first minimum of total energy is at slightly less than 1/3 of a compressional wavelength, which is about
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0.55 of a shear wavelength or 0.59 of a Rayleigh wavelength for this Poisson solid. This minimum of total energy is generally at the same vibrator separation distance as the maximum ratio of P wave to S, or to Rayleigh energy.

![Figure 2: Energy for P, S, and Rayleigh waves. Energy is summed over entire hemisphere below the source, including all azimuths and all angle off-vertical. The abscissa is vibrator separation, scaled to compressional wavelengths. The shaded regions are the P, S, and Rayleigh energy for a triangular source array configuration. Figure from Cassand & Lavergne (1971, 1966).](image)

An early attempted application of these concepts is found in Nyland (1996). This technique used three vibrators in a triangle with separation distance increasing as the size of triangular source array was increased. Recording was done with an arc segment of a circle of phones surrounding the source in the far field. Field implementation of this type test is likely cumbersome. Data analysis was based on looking for the minimum total energy. This is analogous to looking for the minimum in the Cassand and Lavergne figure, but with energy summed only on the surface, including reflections, rather than summing outgoing energy over the full subsurface hemisphere.

### Separating Linear from Non-Linear Effects

For our purposes here, the mutual admittance interactions are considered as non-linear effects. The mathematical / physics theory is based on linear elasticity and linear wave propagation in the far-field, etc. However, the mutual admittance effect is not consistent with linear superposition of multiple sources. Recording vibrator A and then vibrator B separately and then summing will not yield the same result as recording vibrator A and B simultaneously, even neglecting any issues regarding additive noise, etc.

Much of the theoretical basis for mutual admittance interaction of multiple vibrators has been formulated in a fashion such that the results include any linear superposition source array effects, as well as mutual admittance. For example consider the use of a triangular array of vibrators and summing over a range of azimuths as in Cassand and Lavergne (1971, 1966) and Nyland (1996).

However, linear superposition source array effects can be eliminated from field tests of vibrator separations by using a geometry such as seen in figure 3. Note that when considering k-x wavenumber response along the horizontal x-axis of the receiver line, we have a source array that is effectively a point-array for any vibrator separation along the vertical y-axis. For such a simple plane wave analysis, any variation in body wave energy, Rayleigh wave energy, etc. that we record in this geometry will not be due to source array effects, but must rather be due to other factors. With this geometry, data will of course have a constant receiver array response. Of course, deviations from a homogeneous isotropic halfspace that lead to coupling variations, side scattering, etc. will still be potential variables in the data as a function of vibrator separation, and may often be quite significant.

### Field Examples

Utilizing the geometry of figure 3, data were recorded for a suite of vibrator separations, ranging from 5 to 100 meters, with an increment of nominally 5 meters. Analysis of the energies was done in direct fashion in a constant time window centered on a reflector in this good data area, and in a window centered on the ground-roll. Figure 4 shows an example of the ground roll window for one particular vibrator separation.

Figure 5 shows the results of energy variations as a function of the vibrator separation.

Interestingly, the variations in the Rayleigh wave energy roughly mimic the variation seen in the upper shaded zone of the Cassand and Lavergne plot seen in figure 2.

The variations in ground roll energy as a function of the vibrator separation using the geometry of figure 3 are significant. For the ideal half-space model these variations are not linear superposition source array effects. These variations do not seem to be coupling variations given that.
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the energy in the reflected time window doesn’t vary dramatically.

Figure 3: Field Geometry for two vibrator test. Simple concept is that vibrators are separated along the y-axis, orthogonal to the receiver line x-axis. For linear superposition array response, the source array is effectively a point projected to the receiver line.

Further Analysis

Nearly all of the analysis in the past has been done for Poisson solids (0.25). Fig 6 from recent analysis shows the variation in energy for each mode as function of Poisson’s ratio. For more commonly realistic near surface Poisson ratios well above 0.25, the allocation of energy between wave modes is a strong function of Poisson’s ratio, and generally to our detriment. This figure is for 35 Hz with two vibrators spaced at 50 meters. This figure represents total energy radiated at all azimuths and diving angles into the hemisphere under the vibrators.

In contrast to Poisson solid analysis, Fig 7 from recent analysis shows the case of a Poisson’s ratio of 0.45. As seen in lower Poisson ratio cases, in general a key factor is that the Rayleigh wave energy has more variation with vibrator separation than body waves.

Time delays in vibrator far-field waves as a function of ground force levels were reported by Martin and Jack (1990). Meunier (2010) discloses a technique to suppress ground roll using the difference in these time delays for ground roll vs. the time delay for body waves. His technique uses different ground force levels which have different relative time delays. These time delays perhaps depend on non-linear elasticity, with effective rock/soil properties in the near field of the vibrators varying as a function of the ground force level. In contrast, note that mutual admittance effects that cause variations in the energy in each wave type are based on linear elasticity.

In figures 4 and 5 we showed analysis of the relative energy in different wave types based on time windowing of different wave types. Alternatively, particularly in poor data quality areas, we can also consider total energy over large time and offset windows. This is based on the general concept that the minimum in total energy largely coincides with the maximum ratio of compressional body wave to Rayleigh wave.

To advance the overall understanding of the effects discussed in this paper, there is a need to perform simple tests such as in figures 3 and 4 here in more field locations. There is also a need to measure and analyze the near field around interacting vibrators in more detail. Ideally this should include arrays of downhole receivers, and arrays of 3-C and 3-theta rotational phones on the surface around interacting vibrators.

How to use it?

We have shown that there is a theoretical basis and field data evidence for variation in the amount of Rayleigh wave energy being emitted from vibrator arrays that is beyond
linear superposition concepts. To exploit this in practice for surveys requires considering some additional theory and doing some field tests that are beyond current conventional practice.

Many 3D and 4D surveys utilize geometries with crossed source and receiver lines. Commonly vibrators are deployed inline. The effects discussed here indicate that there is some effect generally orthogonal to the source array that may help suppress ground roll.

In routine practice, the test geometry in Fig. 3 and the analysis described here such as in Fig. 4 could quickly and cheaply be employed to assess the importance and behavior of interactions of vibrators.

Some large scale 3D surveys have utilized multiple sets of vibrators, including various techniques such as slip-sweep, simultaneous synchronous sources, etc.. Typically such surveys have relatively rich azimuth and offset distributions; and have effectively high fold, particularly when considering pre-stack imaging. Some such surveys have been acquired with N individual vibrators working independently. The effects described in this paper indicate that a configuration of N/2 pairs of vibrator would perhaps yield an overall better suppression of ground roll over all azimuths.

Numerical modeling of arrays in survey design should include options to include mutual admittance effects. Inline 1D vibrator source arrays have, in fact, 2D array response with significant azimuthal variation, so some usual analysis and displays will need enhancements to incorporate these effects. Mutual admittance based source array modeling can also separately model wave modes, and ratios. Modeling could be generalized to 3D source arrays where we consider the ratios of desired body waves in preferred azimuths and diving angles off-vertical relative to the amount of ground roll seen on the surface.

Conclusions

There is a long known theoretical basis for mutual admittance effects between vibrators that indicates significant effects, particularly on Rayleigh wave energy emitted. There is a simple field test to consider linear superposition and non-superposition effects separately. Some field data seems compatible with the concept that linear superposition array theory does not explain a significant part of what is seen in the field. There is a need to gain more industry experience regarding how consistent and significant these effects are in many varied near surface conditions; and to improve fundamental understanding.

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